

Enhancing entanglement, local and non-local information of accelerated two-qubit and two- qutrit systems via weak-reverse measurements

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Abstract

The possibility of recovering and protecting the entanglement of accelerated 2-qubit and 2-qutrit systems is discussed using weak-reverse measurements. The accelerated partial entangled states are more responsive to be protected than the accelerated maximum entangled states. The accelerated coded local information in qutrit system is more robust than that encoded in a 2-qubit system, and it can be conserved even for larger values of accelerations. Meanwhile, the non-accelerated information in qubit system is not affected by the local operation compared with that depicted on qutrit system. For larger values of accelerations, the weak-reverse measurements can improve the coherent information at the expense of the accelerated information.

Keyword: non-inertial frames, Entanglement, acceleration.

1 Introduction

There are some techniques that have been used to recover the losses of entanglement and protect it from decoherence. Among of these techniques is the quantum purification [1, 2, 3, 4]. Moreno et. al.,[5] showed that, it is possible to improve the coherence by using nested environments. Xiao et. al [6] displayed that the teleportation of Fisher information can be enhanced by using partial measurements. The possibility of purifying and concentrating entanglement by using local filtering is discussed by Yashodamma et. al, [7]. The immunity and protecting the entanglement of accelerated qubit-qutrit system by using the local filtering technique is investigated by Metwally[8]. Recently, it has been shown that the local weak-reverse measurements,(WRM) can be used as a technique to protect and improve the correlation in qubit-qutrit systems [9]. Xiao et. al.[10], showed that, the entanglement losses caused by amplitude damping coherence can be retrieved by using the WRM.

It is well known that, the accelerated systems lose some of its correlations, and consequently, their efficiencies to perform quantum information tasks decrease[11, 12, 13]. This decay depends on the value of the acceleration and the dimensional of the accelerated system[14]. Therefore we are motivated to investigate the possibility of using the weak-reverse measurement's technique to improve some properties of accelerated system. The main task of this contribution is quantifying the amount of entanglement between the two partners in the presence of weak-reverse measurements, investigating the dynamics of the accelerated/non accelerated information (which is encoded on the accelerated/non accelerated particle) and the coherent information between the partners.

The outline of this manuscript is described as follows: The suggested protocol is described in Sec.2. The suggested system and its analytical solutions are given in Sec.3. The effect of the weak-reverse'strengths on the degree of entanglement is discussed in Sec.4., where different initial states settings are considered. The dynamics of the accelerated, non-accelerated and coherent information is investigated in Sec.5. Finally, a conclusion is given in Sec.6.

2 Model

Our idea is to consider two partners, Alice and Bob, who share a two-qubit or two-qutrit systems. These systems are initially prepared in maximum or partial entangled states. Here, we consider only Alice's particle is accelerated, which may be a qubit or a qutrit. The partners decided in advance to use the weak-reverse measurement to protect their quantum communication channel. In the following steps, we describe the proposed protocol:

1. Weak measurements:

Here, both users Alice and Bob perform the weak measurements on their own particles, either qubit or qutrit. We assume that, the two partners share initially a state defined by ρ_{ab}^{ini} . After performing the weak measurements, the partners share a new state defined by ρ_{ab}^{out} ,

$$\rho_{ab}^{out} = \mathcal{W}_j \mathcal{W}_j (\rho_{ab}^{ini}) \mathcal{W}_j^\dagger \mathcal{W}_j^\dagger, \quad i, j = q, t, \quad (1)$$

where \mathcal{W}_i are the weak measurements which are defined by the following operators [15, 16]

$$\begin{aligned} \mathcal{W}_q &= |0\rangle\langle 0| + \sqrt{1 - \alpha_q^{(\ell)}} |1\rangle\langle 1| \quad (\text{for qubit}), \\ \mathcal{W}_t &= |0\rangle\langle 0| + \sqrt{1 - \alpha_t^{(1)}} |1\rangle\langle 1| + \sqrt{1 - \alpha_t^{(2)}} |2\rangle\langle 2| \quad (\text{for-qutrit}), \end{aligned} \quad (2)$$

and $\alpha_q^{(\ell)}, \alpha_t^{(\ell)}, \ell = 1, 2$ are the strengths of the weak measurements of the qubit and the qutrit, respectively.

2. Acceleration step:

It is assumed that, only Alice' particle is moving with a uniform acceleration meanwhile Bob's partial is assumed to be inertial. If the shared particles are of fermions types, then in the Minkowski frame the qubit system is transformed in the Rindler frame as,

$$|0_M\rangle = \cos r |0\rangle_I |0\rangle_{II} + \sin r |1\rangle_I |1\rangle_{II}, \quad |1_M\rangle = |1\rangle_I |0\rangle_{II} \quad (3)$$

while, for the qutrit system, the vacuum, the spin up and spin down states are transformed into Rindler space as,

$$\begin{aligned} |0_M\rangle &= \cos^2 r |0\rangle_I |0\rangle_{II} + e^{i\phi} \sin r \cos r (|\mathcal{U}\rangle_I |\mathcal{D}\rangle_{II} + |\mathcal{D}\rangle_I |\mathcal{U}\rangle_{II}) + e^{2i\phi} \sin^2 r |\mathcal{D}\rangle_I |\mathcal{P}\rangle_{II}, \\ |\mathcal{U}_M\rangle &= \cos r |\mathcal{U}\rangle_I |0\rangle_{II} + e^{i\phi} \sin r |\mathcal{P}\rangle_I |\mathcal{U}\rangle_{II}, \\ |\mathcal{D}_M\rangle &= \cos r |\mathcal{D}\rangle_I |0\rangle_{II} - e^{i\phi} \sin r |\mathcal{P}\rangle_I |\mathcal{D}\rangle_{II}, \end{aligned} \quad (4)$$

where $|\mathcal{U}\rangle, |\mathcal{D}\rangle$ and $|\mathcal{P}\rangle$ are the spin up, spin down and pair states, respectively. The acceleration r is defined such that $\tan r = \text{Exp}[-\pi\omega \frac{c}{a}]$, $0 \leq r \leq \pi/4$, $-\infty \leq a \leq \infty$, ω is the frequency, c is the speed of light and ϕ is the phase space which can be absorbed in the definition of the operators [17]. After tracing the particle in the second region (II), the final state represents the accelerated quantum channel between Alice and Bob, ρ_{ab}^{acc} .

3. Reverse measurement step:

In this step, the users apply the reverse measurement operations on the accelerated state ρ_{ab}^{acc} to obtain the final state as,

$$\rho_{ab}^{Final} = \mathcal{R}_i \mathcal{R}_i (\rho_{ab}^{acc}) \mathcal{R}_i^\dagger \mathcal{R}_i^\dagger, \quad i = q, t, \quad (5)$$

where

$$\begin{aligned}\mathcal{R}_q &= \sqrt{1 - \beta_q^{(\ell)}} |0\rangle\langle 0| + |1\rangle\langle 1|, \quad \ell = 1, 2 \\ \mathcal{R}_t &= \sqrt{(1 - \beta_t^{(1)})(1 - \beta_t^{(2)})} |0\rangle\langle 0| + \sqrt{1 - \beta_t^{(1)}} |1\rangle\langle 1| + \sqrt{1 - \beta_t^{(2)}} |2\rangle\langle 2|, \quad (6)\end{aligned}$$

and $\beta_q^{(\ell)}, \beta_t^{(\ell)}, \ell = 1, 2$ are the strengths of the reverse measurement operations. Some properties of the final state Eq.(5), such as the behavior of entanglement, accelerated local non-local information are examined. Particularly, the effect of the initial state settings and the local operations strengths on these properties.

2.1 The suggested Systems

1. Two-qubit system

In this section, it is assumed that the partners, Alice and Bob, share a partially entangled state, which is called X -state. This state can be written, using the computational basis as,

$$\begin{aligned}\rho_x &= \mathcal{B}_1 (|00\rangle\langle 00| + |11\rangle\langle 11|) + \mathcal{B}_2 (|00\rangle\langle 11| + |11\rangle\langle 00|) \\ &+ \mathcal{B}_3 (|01\rangle\langle 01| + |10\rangle\langle 10|) + \mathcal{B}_4 (|01\rangle\langle 10| + |10\rangle\langle 01|)\end{aligned} \quad (7)$$

where $\mathcal{B}_1 = \frac{1}{4}(1 + c_{33})$, $\mathcal{B}_2 = \frac{1}{4}(c_{11} + c_{22})$, $\mathcal{B}_3 = \frac{1}{4}(1 - c_{33})$, and $\mathcal{B}_4 = \frac{1}{4}(c_{11} - c_{22})$, and $c_i, i = 1, 2, 3$ are the diagonal elements of 3×3 cross dyadic. The state (7) can be described by its Bloch vectors, $\vec{s}_j = 0, j = 1, 2$ and a dyadic ${}^4\vec{C}$ with $c_{ij} = 0$ for $i \neq j$. From this state, one can get maximum entangled classes, MES. For example, if we set $c_{11} = c_{22} = c_{33} = -1$, one gets the singled state, $\rho_{\psi^-} = |\psi^-\rangle\langle\psi^-|$. The Werener state can be obtained if we set $c_{11} = c_{22} = c_{33} = -x$, etc. As mentioned earlier, only Alice's particle will be accelerated [18]. Due to the acceleration the degree of entanglement decreases. Therefore, the partners try to improve the degree of entanglement by applying the weak-reverse measurements. At the end of the protocol, the partners share the state.

$$\begin{aligned}\rho_{ab}^{Final} &= \frac{1}{\mathcal{N}_q} \left\{ \mathcal{B}_1 |00\rangle\langle 00| + \tilde{\mathcal{B}}_2 |00\rangle\langle 11| + \tilde{\mathcal{B}}_3 |01\rangle\langle 01| + \tilde{\mathcal{B}}_4 |01\rangle\langle 10| \right. \\ &+ \left. \mathcal{B}_5 |10\rangle\langle 10| + \tilde{\mathcal{B}}_6 |10\rangle\langle 01| + \tilde{\mathcal{B}}_7 |11\rangle\langle 11| + \tilde{\mathcal{B}}_8 |11\rangle\langle 00| \right\}\end{aligned} \quad (8)$$

where,

$$\begin{aligned}\tilde{\mathcal{B}}_1 &= c_1^2 \mathcal{B}_1 (1 - \beta_q^{(1)})(1 - \beta_q^{(2)}), \\ \tilde{\mathcal{B}}_2 &= c_1 \mathcal{B}_2 \sqrt{1 - \beta_q^{(1)}} \sqrt{1 - \beta_q^{(2)}} \sqrt{1 - \alpha_q^{(1)}} \sqrt{1 - \alpha_q^{(2)}}, \\ \tilde{\mathcal{B}}_3 &= c_1^2 \mathcal{B}_3 (1 - \beta_q^{(1)})(1 - \alpha_q^{(2)}), \\ \tilde{\mathcal{B}}_4 &= c_1 \mathcal{B}_4 \sqrt{1 - \beta_q^{(1)}} \sqrt{1 - \beta_q^{(2)}} \sqrt{1 - \alpha_q^{(1)}} \sqrt{1 - \alpha_q^{(2)}}, \\ \tilde{\mathcal{B}}_5 &= (1 - \beta_q^{(2)}) [s_1^2 \mathcal{B}_1 + (1 - \alpha_q^{(1)}) \mathcal{B}_3], \quad \tilde{\mathcal{B}}_6 = \tilde{\mathcal{B}}_4 \\ \tilde{\mathcal{B}}_7 &= (1 - \alpha_q^{(1)})(1 - \alpha_q^{(2)}) \mathcal{B}_1, \quad \tilde{\mathcal{B}}_8 = \tilde{\mathcal{B}}_2,\end{aligned} \quad (9)$$

and $\mathcal{N}_q = \tilde{\mathcal{B}}_1 + \tilde{\mathcal{B}}_3 + \tilde{\mathcal{B}}_5 + \tilde{\mathcal{B}}_6$ is the normalization factor and $c_1 = \cos r, s_1 = \sin r$.

2. Two-qutrit system:

In this section, we investigate the effect of the local-weak-reverse operations on a system consists of two qutrits, where it is assumed that only Alice's qutrit will be accelerated. In the computational basis, the initial state of this system can be written as [19],

$$|\psi_t\rangle = \frac{1}{\sqrt{2+\gamma^2}} (|00\rangle + |11\rangle + \gamma|22\rangle), \quad (10)$$

where it turns into a maximum entangled state for $\gamma = 1$. The users apply the protocol which is described in Sec.2. At the end the partners share the following state,

$$\begin{aligned} \rho_{ab}^{Final} = & \frac{1}{\mathcal{N}_t} \left\{ \mathcal{D}_1|00\rangle\langle 00| + \mathcal{D}_2|00\rangle\langle 11| + \mathcal{D}_3|10\rangle\langle 10| + \mathcal{D}_4|11\rangle\langle 00| + \mathcal{D}_5|11\rangle\langle 11| \right. \\ & + \mathcal{D}_6|20\rangle\langle 20| + \mathcal{D}_7|22\rangle\langle 00| + \mathcal{D}_8|22\rangle\langle 11| + \mathcal{D}_9|22\rangle\langle 22| + \mathcal{D}_{10}|00\rangle\langle 22| \\ & \left. + \mathcal{D}_{11}|11\rangle\langle 22| \right\}, \end{aligned} \quad (11)$$

where

$$\begin{aligned} \mathcal{D}_1 &= c_1^2 \mathcal{R}_{00}^2 \mathcal{A}_1, & \mathcal{D}_2 &= c_1^3 \mathcal{R}_{00} \mathcal{R}_{11} \mathcal{A}_2, & \mathcal{D}_3 &= c_1^2 s_1^2 \mathcal{R}_{10}^2 \mathcal{A}_1, \\ \mathcal{D}_4 &= c_1^3 \mathcal{R}_{00} \mathcal{R}_{11} \mathcal{A}_4, & \mathcal{D}_5 &= c_1^3 \mathcal{R}_{11}^2 \mathcal{A}_5, & \mathcal{D}_6 &= c_1^2 s_1^2 \mathcal{R}_{20}^2 \mathcal{A}_1, \\ \mathcal{D}_7 &= c_1^3 \mathcal{R}_{22} \mathcal{R}_{00} \mathcal{A}_7, & \mathcal{D}_8 &= c_1^2 \mathcal{R}_{22} \mathcal{R}_{11} \mathcal{A}_8, & \mathcal{D}_9 &= c_1^2 \mathcal{R}_{22}^2 \mathcal{A}_9, \\ \mathcal{D}_{10} &= \mathcal{R}_{00} \mathcal{R}_{22} \mathcal{A}_3 c^3, & \mathcal{D}_{11} &= \mathcal{R}_{11} \mathcal{R}_{22} \mathcal{A}_6 c^2, \end{aligned} \quad (12)$$

The normalization factor is given by $\mathcal{N}_t = \mathcal{D}_1 + \mathcal{D}_3 + \mathcal{D}_5 + \mathcal{D}_6 + \mathcal{D}_9$ and the coefficients $\mathcal{A}_i, i = 1..9$ are given by,

$$\begin{aligned} \mathcal{A}_1 &= \frac{1}{2+\gamma^2}, & \mathcal{A}_2 &= \frac{\sqrt{1-\alpha_t^{(1)}}\sqrt{1-\alpha_t^{(2)}}}{2+\gamma^2}, & \mathcal{A}_3 &= \frac{\gamma\sqrt{1-\alpha_t^{(1)}}\sqrt{1-\alpha_t^{(2)}}}{2+\gamma^2}, \\ \mathcal{A}_4 &= \mathcal{A}_2, & \mathcal{A}_5 &= (2+\gamma^2)\mathcal{A}_2^2, & \mathcal{A}_6 &= \gamma\mathcal{A}_2\sqrt{1-\alpha_t^{(1)}}\sqrt{1-\alpha_t^{(2)}}, \\ \mathcal{A}_7 &= \frac{\gamma}{2+\gamma^2}\sqrt{1-\alpha_t^{(1)}}\sqrt{1-\alpha_t^{(2)}}, & \mathcal{A}_8 &= \frac{\gamma(1-\alpha_t^{(1)})}{2+\gamma^2}\sqrt{1-\alpha_t^{(2)}}\sqrt{1-\alpha_t^{(2)}}, \\ \mathcal{A}_9 &= \frac{\gamma^2}{2+\gamma^2}(1-\alpha_t^{(1)})(1-\alpha_t^{(2)}), \end{aligned}$$

and $\mathcal{R}_{ij}, i, j = 0, 1, 2$ are given by

$$\begin{aligned} \mathcal{R}_{00} &= \sqrt{(1-\beta_1^{(1)})(1-\beta_2^{(1)})}\sqrt{(1-\beta_1^{(2)})(1-\beta_2^{(2)})}, \\ \mathcal{R}_{01} &= \sqrt{1-\beta_1^{(2)}}\sqrt{(1-\beta_1^{(1)})(1-\beta_2^{(1)})}, \\ \mathcal{R}_{02} &= \sqrt{1-\beta_2^{(2)}}\sqrt{(1-\beta_1^{(1)})(1-\beta_2^{(1)})}, \\ \mathcal{R}_{10} &= \sqrt{1-\beta_1^{(1)}}\sqrt{(1-\beta_1^{(2)})(1-\beta_2^{(2)})}, \\ \mathcal{R}_{11} &= \sqrt{1-\beta_1^{(1)}}\sqrt{1-\beta_1^{(2)}}, & \mathcal{R}_{12} &= \sqrt{1-\beta_1^{(1)}}\sqrt{1-\beta_2^{(2)}}, \\ \mathcal{R}_{20} &= \sqrt{1-\beta_2^{(1)}}\sqrt{(1-\beta_1^{(2)})(1-\beta_2^{(2)})}, & \mathcal{R}_{22} &= \sqrt{1-\beta_2^{(1)}}\sqrt{1-\beta_2^{(2)}}. \end{aligned}$$

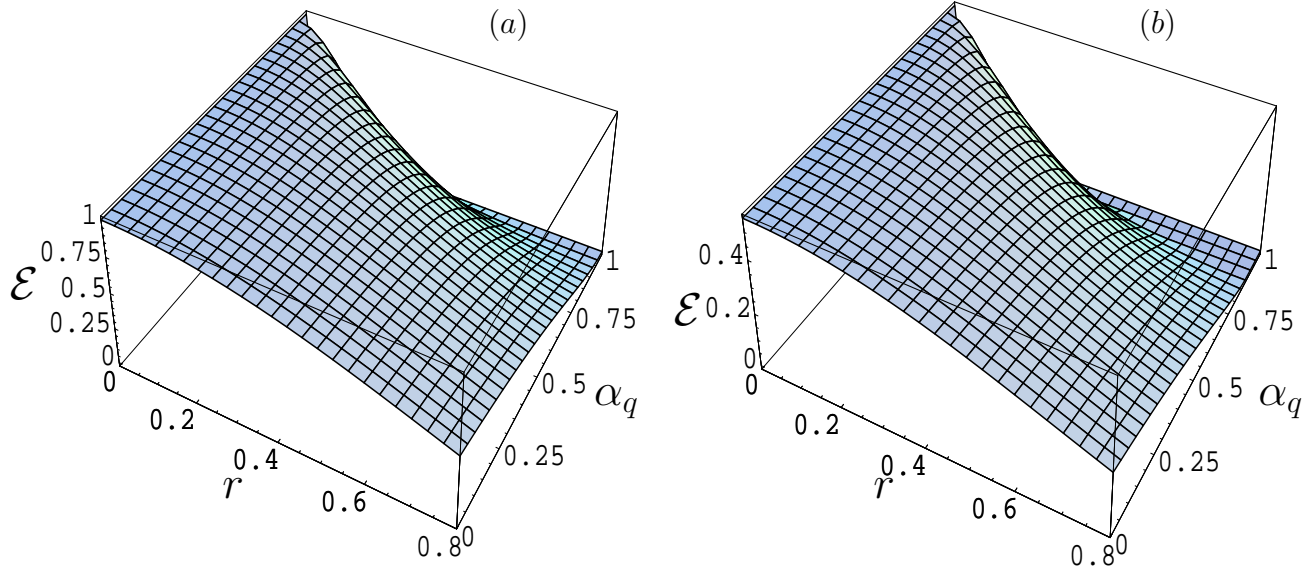


Figure 1: The entanglement of the final state of the two-qubit system (8) where we set $\alpha_q^{(1)} = \alpha_q^{(2)} = \beta_q^{(1)} = \beta_q^{(2)} = \alpha_q$. (a) The initial state is prepared in a maximum entangled states, i.e., ($c_{11} = c_{22} = c_{33} = 1$). (b) The initial state is prepared in a partial entangled state (Werner state, $c_{11} = c_{22} = c_{33} = 0.7$),

3 Entanglement

To quantify the survival degree of entanglement \mathcal{E} contained in the system, we use the negativity as a measure. The negativity is defined as,

$$\mathcal{E} = \max \left(0, \sum_i \lambda_i \right), \quad (13)$$

where λ_i , are the eigenvalues of the partial transpose of $\rho_{ab}^{T_a}$ [20].

Fig.(1a), displays the behavior of entanglement of the final state (8), where it is assumed that, the partners share initially a singlet state i.e., $c_{11} = c_{22} = c_{33} = 1$ and we have set $\alpha_q^{(1)} = \alpha_q^{(2)} = \beta_q^{(1)} = \beta_q^{(2)} = \alpha_q$. It is clear that, for $r = 0$ the degree of entanglement is maximum i.e., ($\mathcal{E} = 1$). The general behavior shows that, the entanglement decreases as the acceleration parameter, r increases. On the other hand, as one increases the strengths of the local operations, the entanglement increases. However, for further values of $\alpha \in [0.75, 1]$, the entanglement decreases.

The behavior of entanglement of a system initially prepared in a partially entangled state of Werner type is displayed in Fig.(1b). The general behavior is similar to that for the MES (Fig.(1a)), but with smaller decay rate. On the other hand, for $\alpha \in [0.75, 1]$, the entanglement re-appears again while, it vanishes completely for the MES.

The dynamics of entanglement under the effect of the local weak-reverse measurement for a system is initially prepared in a two-qutrit system is described in Fig.(2). The general behavior is similar to that predicted in Figs.(1), where the entanglement decays as r increases. However, the upper and lower bounds of entanglement depend on initial state settings. It is clear that, for the two-qutrit system the entanglement increases slowly compared with that shown in Fig.(1). Also, as displayed in Fig.(2b), the increasing rate of entanglement for PES is larger than that shown for MES.

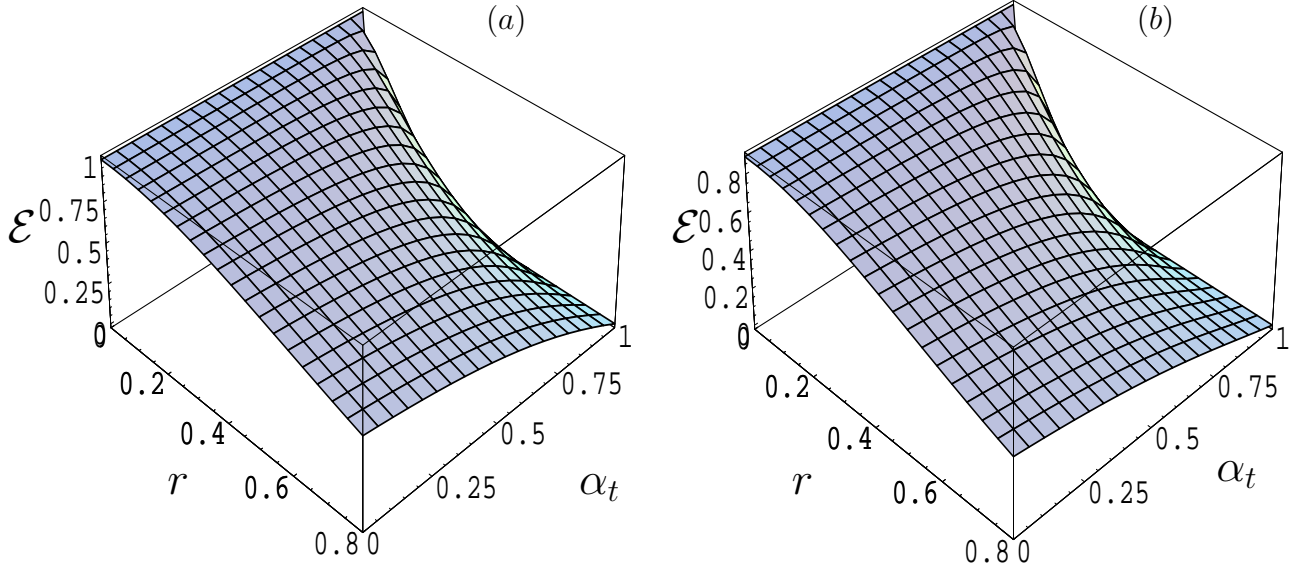


Figure 2: The of entanglement, of the two-qutrit system Eq.(11), where (a) maxim entangled state ($\gamma = 1$) and (b) partial entangled state i.e., ($\gamma = 0.5$), where set $\alpha_t^{(1)} = \alpha_t^{(2)} = \beta_t^{(1)} = \beta_t^{(2)} = \alpha_t$.

From the previous figures, one concludes that the weak and reverse measurements can recover the loss of entanglement for small values of accelerations as the local operations' strengths increase for different intervals. The length of these intervals depends on the initial states settings: two-qubit, or two qutrits, maximum / partial entangled states. The increasing rate of entanglement is much larger for systems that are initially prepared in partial entangled states. The partially entangled 2- qubits state is more robust than the partially entangled two-qutrit state.

4 Local and Non-local Information

4.1 Two-qubit system

In this subsection, we quantify Alice's and Bob's information, which represents the accelerated (\mathcal{I}_a) and non-accelerated, (\mathcal{I}_b) information, respectively. Moreover, the amount of the non-local information between the partners defined by the coherent information (\mathcal{I}_{coh}) will be quantified also. In an explicit form, theses three types of information can be written as,

$$\begin{aligned}
 \mathcal{I}_a &= -\frac{\tilde{\mathcal{B}}_1 + \tilde{\mathcal{B}}_3}{\mathcal{N}_q} \log \left(\frac{\tilde{\mathcal{B}}_1 + \tilde{\mathcal{B}}_3}{\mathcal{N}_q} \right) - \frac{\tilde{\mathcal{B}}_5 + \tilde{\mathcal{B}}_7}{\mathcal{N}_q} \log \left(\frac{\tilde{\mathcal{B}}_5 + \tilde{\mathcal{B}}_7}{\mathcal{N}_q} \right) \\
 \mathcal{I}_b &= -\frac{\tilde{\mathcal{B}}_1 + \tilde{\mathcal{B}}_5}{\mathcal{N}_q} \log \left(\frac{\tilde{\mathcal{B}}_1 + \tilde{\mathcal{B}}_5}{\mathcal{N}_q} \right) - \frac{\tilde{\mathcal{B}}_3 + \tilde{\mathcal{B}}_7}{\mathcal{N}_q} \log \left(\frac{\tilde{\mathcal{B}}_3 + \tilde{\mathcal{B}}_7}{\mathcal{N}_q} \right) \\
 \mathcal{I}_{coh} &= \sum_{i=1}^4 \mu_i \log(\mu_i),
 \end{aligned} \tag{14}$$

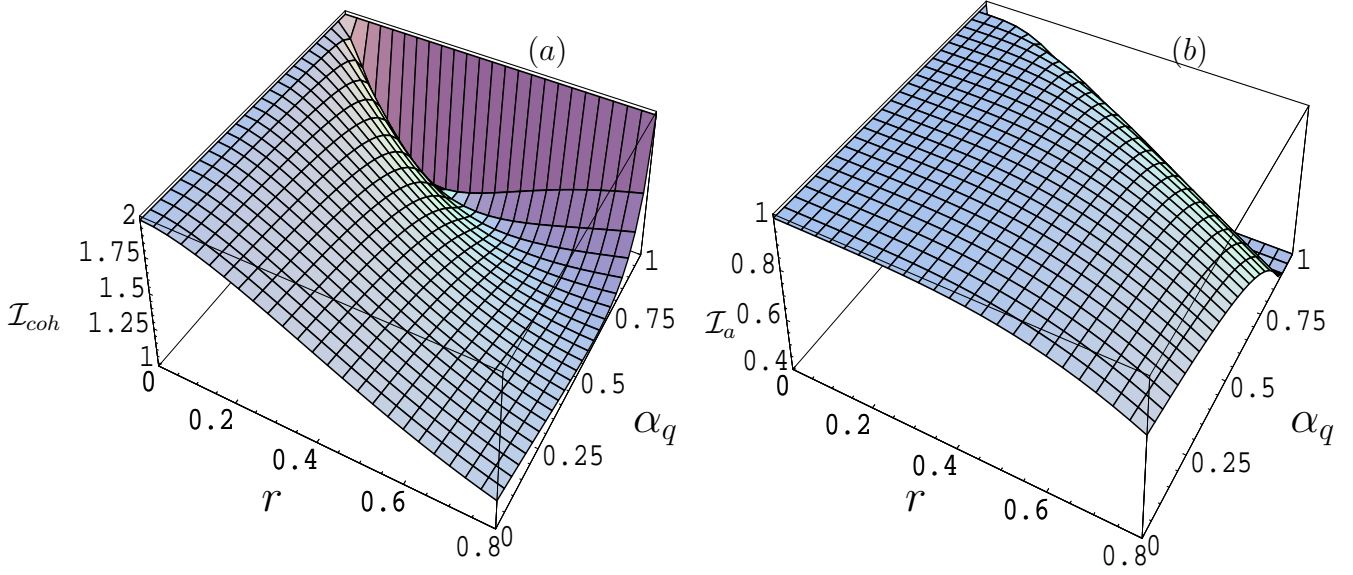


Figure 3: The local and non-local information for a system is initially prepared in a MES of two-qutrit system. (a) The coherent information between Alice and Bob, \mathcal{I}_{coh} and (b) the accelerated local information \mathcal{I}_a .

where

$$\begin{aligned}\mu_{1,2} &= \frac{1}{2\mathcal{N}_q} \left\{ (\tilde{\mathcal{B}}_1 + \tilde{\mathcal{B}}_7) \pm \sqrt{(\tilde{\mathcal{B}}_1 - \tilde{\mathcal{B}}_7)^2 + 4\tilde{\mathcal{B}}_2\tilde{\mathcal{B}}_8} \right\}, \\ \mu_{3,4} &= \frac{1}{2\mathcal{N}_q} \left\{ (\tilde{\mathcal{B}}_3 + \tilde{\mathcal{B}}_5) \pm \sqrt{(\tilde{\mathcal{B}}_3 - \tilde{\mathcal{B}}_5)^2 + 4\tilde{\mathcal{B}}_4\tilde{\mathcal{B}}_6} \right\},\end{aligned}$$

Fig.(3) displays the behavior of the coherent information and the accelerated local information, where we set $\alpha_q^{(1)} = \alpha_q^{(2)} = \beta_q^{(1)} = \beta_q^{(2)} = \alpha_q$. At zero acceleration, both types of information are maximum. As Alice's particle is accelerated, the coherent information decreases. However, for a fixed value of r , the coherent information increases as the strengths of the local operations increase. Meanwhile, the accelerated local information increases at the expense of the coherent information. For larger values of α_q , the accelerated information vanishes while the coherent information increases. This coherent information, no longer represents quantum information but it describes a classical information, because the two particles are almost separable for larger values of α_q as displayed in Fig.(1).

In Fig.(4), we scrutinize the effect of the WRM on the local and non local information, as well as on the coherent information. The behavior of the accelerated information \mathcal{I}_a and the non-accelerated information \mathcal{I}_b are described in Fig.(4a), where it is assumed that, the system is either initially prepared in a maximum entangled state (MES) or in a partial entangled state (PES). At zero acceleration ($r = 0$) the information which is encoded in Alice's and Bob's qubit is maximum i.e. $\mathcal{I}_a = \mathcal{I}_b = 1 \text{ bit}$. For small values of $r \in [0, 2.5]$, \mathcal{I}_a and \mathcal{I}_b are remaining maximum (1 bit). For larger values of r , the accelerated information \mathcal{I}_a decreases as the acceleration r increases, while Bob's information (non-accelerated information) is slightly affected due to the local WRM. Moreover, the decay rate of the accelerated information for a system prepared initially in a partial entangled state is smaller than that displayed for systems that are initially prepared in MES. This shows that, the possibility of protecting the accelerated local information encoded in PES by the WRM is much better

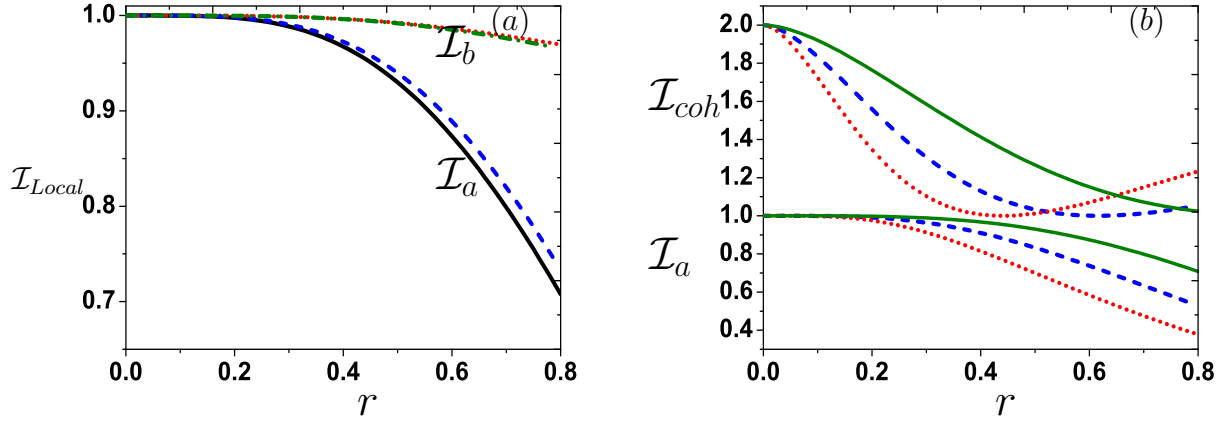


Figure 4: (a) Alice and Bob's information for a system is initially prepared in a MES or PES. The solid and dash curves represent Alice's for system is initially prepared in MES and PES, respectively, while the dash-dot and dot curves represent Bob's local information for MES and PES respectively, where we set $\alpha_q^{(1)} = \alpha_q^{(2)} = \beta_q^{(1)} = \beta_q^{(2)} = \alpha_q = 0.5$ (b) The local and non local information for different values of $\alpha_q = 0.5, 0.8, 0.9$, for the solid, dash and dot curves respectively.

than protecting that coded in MES.

The effect of the local WRM on the accelerated local information, \mathcal{I}_a and the coherent information \mathcal{I}_{coh} is depicted in Fig.(4b), where it is assumed that the system is initially prepared in the MES. In these calculations, we consider that the strengths of the local weak measurement and the reverse measurements are equal ($\alpha_q = 0.5$). The behavior of information shows that, both types of the information are maximum at $r = 0$. As r increases, the accelerated local information \mathcal{I}_a has maximum values for $r \in [0, 2.5]$, while the coherent information decreases for any value of $r > 0$. It is clear that, as the strengths of the WRM increase, both types of information decrease. However the decay rate of \mathcal{I}_{coh} is much larger than that for accelerated local information, \mathcal{I}_a . For larger values of r and α_q , the coherent information increases at the expense of the local accelerated information.

From these figures, one concludes that the accelerated information for small values of acceleration can be protected by using the weak-reverse measurement. The possibility of protecting the accelerated information which is encoded in partial entangled states is larger than that encoded in a maximum entangled states. The local measurement has a very slight effect on the non-accelerated local information. For larger values of accelerations, the coherent information increases at the expense of the accelerated information

4.2 Two-qutrit system

Fig.(5) shows the behavior of the coherent information \mathcal{I}_{coh} and the accelerated information \mathcal{I}_a for a system initially prepared in an entangled maximum two-qutrit system. It is clear that, both types of information decrease as the acceleration r increases. The decay rate is larger for the coherent information compared with that depicted for the accelerated information. However, as the strengths of the local operations increase both types of information are improved. For $\alpha_t \in [0.8, 1]$, the coherent information increases at the expense of the accelerated information.

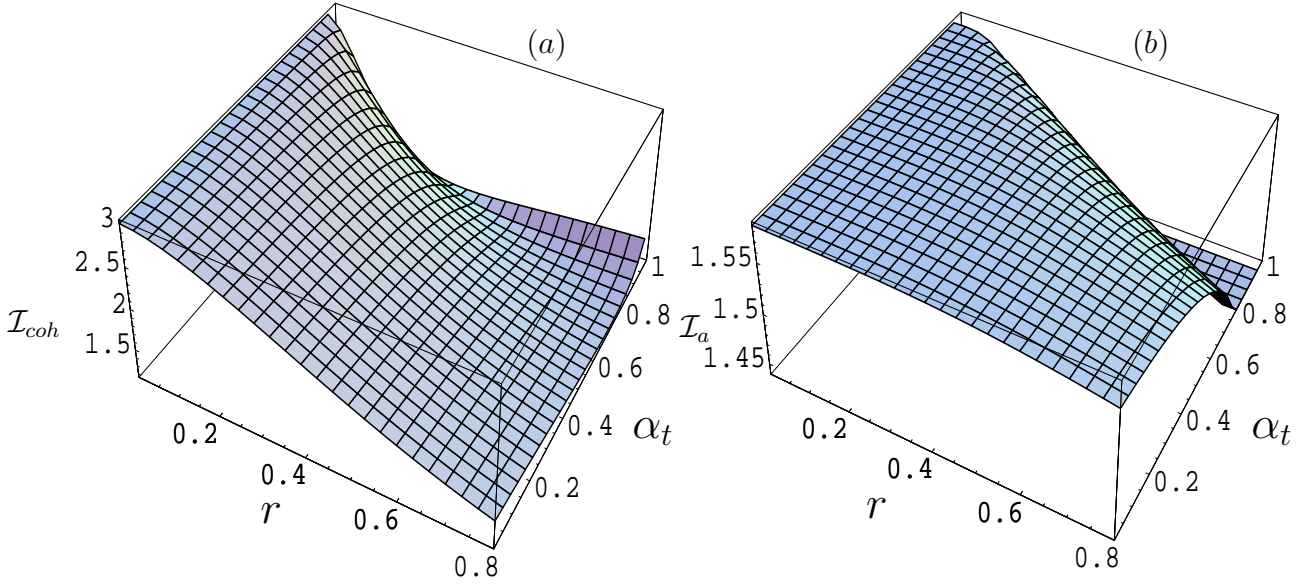


Figure 5: The same as Fig.(3), but the system is initially prepared in a maximum entangled 2-qutrits state.

The dynamics of different types of information coded in a system initially prepared in a maximum two-qutrit entangled state is described in Fig.(6). The behavior of the accelerated local information \mathcal{I}_a and the non-accelerated local information, \mathcal{I}_b is displayed in Fig.(6a), where it is assumed that, the system initially prepared in MES or PES. It is clear that, the accelerated information for a system is initially prepared in a MES is almost stable and it is a maximum for any value of r . On the other hand, starting from PES, the initial values of the accelerated information is smaller than that depicted for MES. However, the upper bounds of \mathcal{I}_a increase as the acceleration r increases. These upper bounds become larger than that depicted for MES for $r \in [0, 0.8]$. For non-accelerated information the behavior is completely different from those shown for the two-qubit system, where it is always smaller than the accelerated information. Moreover, it decreases as r increases.

Fig(6b) shows the behavior of the coherent information and the accelerated information for different values of the local operation's strengths, with initial system prepared in a MES of the two-qutrit system. It is clear that, \mathcal{I}_{coh} decreases as α_t and r increase. For smaller values of α_t , the coherent information decreases faster than that displayed for larger values of α_t . On the other hand, for larger values of α_t and r , the coherent information increases to reach its upper bounds as $r \rightarrow \infty$. Also, the general behavior of the accelerate information shows that, \mathcal{I}_a decreases as the strengths of the local operations increases. However, the decay rate is smaller than the coherent information. This explain the decay of the non-accelerated information displayed in Fig.(6a).

From these figures one can extract some important facts. The local information which is encoded in the accelerated part of the composite system can be protect by using the WRM. The local accelerated information which is encoded in a partial entangled state can be improved even for larger accelerations. The non-accelerated local information is very sensitive to the local operations (WRM), compared with that shown for the two-qubit system. The decay rate of accelerated information is much smaller than that shown for the coherent information. For larger values of accelerations, the coherent information increases at the expense of the non-accelerated local information.

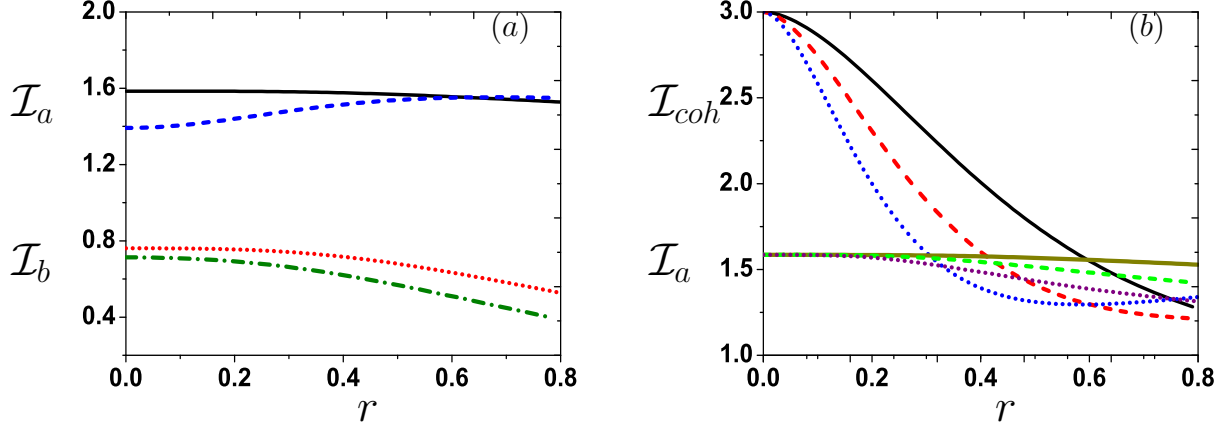


Figure 6: Local and non-local information of a MES of two-qutrit system where $\alpha_t = 0.5, 0.8, 0.9$.

5 Conclusion

In this contribution, we assume that two users, Alice and Bob, share 2-qubit or 2-qutrit states as a communication channels to perform some quantum information tasks. Alice is allowed to accelerate her qubit/qutrit. Subsequently, the coherence of the communication channel between the partners decreases. Thus, we try to investigate the possibility of utilizing the weak-reverse technique to improve the quantum correlations between the users. It is shown that, the initial state settings, the values of the acceleration and the strengths of the weak-reverse operations represent control keys to the behavior of entanglement, local and non-local information.

It is demonstrated that, for small values of acceleration the improving rate of the quantum correlation of the communication channels increases as the strengths of the weak-reverse operation increase. However, as one increases the local operations' strengths further the correlation decreases. Our results show that, starting from less entangled state the improving rate of correlations is better than that depicted for maximum entangled states. On the other hand, the increasing rate of the correlation for the two-qutrit system is larger for large values of the acceleration comparing with that displayed for the two-qubit-system.

Additionally, the behavior of the local information which includes the accelerated and non-accelerated information is assayed. It is shown that, for the two-qubit system, the non-accelerated information is almost stable and slightly decreases for systems initially prepared in maximum entangled state. But the accelerated information decreases as the acceleration increases. Moreover, the encoded information in a system initially prepared in partial entangled state is more robust than that encoded in a system initially prepared in maximum entangled states. Theses results are dramatically changed for the two-qutrit systems, where the non-accelerated information is more fragile than the accelerated information. Also, the encoded information in the partial entangled state of the two-qutrit system is improved as the acceleration increases under weak-reverse measurements.

Finally, the general behavior of the coherent information displays that. it decreases as the acceleration increases. For a fixed value of acceleration, the coherent information increases as the strength of the weak-reverse measurement increases. However, for the two-qubit system, larger values of these strengths cause a sudden decay of information for small values of the

accelerations and gradually decay for larger values of these accelerations. Meanwhile, for the two-qutrit system, the coherent information slightly increases at larger values of the weak-reverse measurements' strengths. The changes happened at the expense of the local information.

In conclusion, it is possible to improve and protect the accelerated communication channel by using weak-reverse measurements. The possibility of protecting the local encoded information on an accelerated 2-qutrit system is larger than that displayed for 2-qubit system. The restrained decay rate of the coded information on accelerated partial entangled state is better than that displayed for maximum entangled state. The encoded information in the accelerated part of the two-qutrit system can be improved by using the local operations, while it decays for the two-qubit system. Therefore, to perform quantum key distribution or quantum coding protocols using accelerated system, it is better to use partial entangled 2-qutrit systems with larger accelerations.

References

- [1] C.H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J.A. Smolin, W.K. Wootters, "Purification of Noisy Entanglement and Faithful Teleportation via Noisy Channels", Phys. Rev. Lett. **76** 722 (1996).
- [2] D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, A. Sanpera, "Quantum Privacy Amplification and the Security of Quantum Cryptography over Noisy Channels", Phys. Rev. Lett. **77** 2818 (1996).
- [3] N. Metwally, "More efficient entanglement purification", Phys. Rev. A **66** 054302 (2002).
- [4] N. Metwally, A.-S. Obada, "More efficient purifying scheme via controlled-controlled NOT gate", Physics Letters A **352** 45 (2006).
- [5] H.. J. Moreno, T. Gorin and T. H. Seligman, "Improving coherence with nested environments", Phys. Rev. A. **92** 030104(R) (2015).
- [6] X. Xiao, Y. Yao, Wo-J. Zhong, Y.-L. Li and Y.M. Xie, "Enhancing teleportation of quantum Fisher information by partial measurements", Phys. Rev. A **93** 012307 (2016).
- [7] K. O. Yashodamma, P. J. Geetha and Sudha, "Purification and redistribution of entanglement via single local filtering", Int. J. Quantum Inform. **12** 1450004 (2014).
- [8] N. Metwally "Immunity, Improving and Retrieving the lost entanglement of accelerated qubit-qutrit system via local Filtering", h i arXiv:1603.01429 (2016).
- [9] J.-L. Guo, J.-L. Wi and W. Qin, "Enhancement of quantum correlations in qubit-qutrit system under decoherence of finite temperature", Quantum In Process **14** 1399-1410 (2015).
- [10] X.Xiao, "Protecting qubit-qutrit entanglement from amplitude damping decoherence from amplitude damping decoherence via weak measurement and reversal", Phy. Scr. **89** 065102 (7pp) (2014).
- [11] N. Metwally "Teleportation of accelerated Information" J. Opt. Soci. Am B **30** 233 (2013).

- [12] T. G. Downes, T. C. Ralph and N. Walk," Quantum communication with an accelerated partner", Phys. Rev. A **87** 012327 (2013).
- [13] N. Metwally and A. Sagheer" Quantum coding in non-inertial frames", Quantum Information Processing **13** 771 (2014).
- [14] N. Metwally," Entanglement of simultaneous and non-simultaneous accelerated qubit-qutrit systems", Quantum Inf. and Comput (QIC), **16** 0530-0542 (2016).
- [15] Y. S. Kim, J. C. Lee, O. Kwon, Y. H. Kim,"Protecting entanglement from decoherence using weak measurement and quantum measurement reversal", Nat Phys. **8** 117 (2012).
- [16] X. Xiao, T, L. Li "Protecting qutrit-qutrit entanglement by weak measurement and reversal, Eur. Phys. J. D **67** 204 (2013).
- [17] E. M.-Martinez, I. Fuentes,"Redistribution of particle and antiparticle entanglement in noninertial frames", Phys. Rev. A **83**, 052306 (2011); J. Doukas, E. G. Brown, A. J. Dragan, and R. B. Mann, "Entanglement and discord: Accelerated observations of local and global modes", Phys. Rev. A **87**, 012306 (2013).
- [18] N. Metwally, J. Mod. Phys. B **27** 1350155 (18pages) (2013).
- [19] E. A. Fonseca and F. Parisio," Maesure of nonlocality which is maximal for maximally entangled qutrits", Phys. Rev. A **92** 030101 (R) (2015).
- [20] G. Karpat and Z. Gedik, Phys. Lett. A **375** 4166-4171 (2011).